THE MAGNETIC TRANSLATION AND ROTATION OF FLUID SUSPENSIONS OF PARTICLES REPRESENTATIVE OF CELLS

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Recently, numerous investigators (Barnothy, M.F., 1963) have reported Biomagnetic Effects, many apparently caused by the action of stationai magnetic fields on living organisms. It is instructive to calculate the magneto-mechanical forces on small particles such as might be involved in biological processes, in order to see whether they are the causative mechanism. Models of human red blood cells will be considered in particular, as they are the "most magnetic ordinary tissue component.

First, consider the redistribution of spherical particles of volume V cm radius r cm, volume susceptibility K in a fluid of susceptibility K_o when acted upon by a magnetic field H, oersted, which has a gradient dH/dx. The concentr tion of n(x) particles per cm^3, adapting the equations derived by Einstein (Einst A., 1956), will be

$$n/n_o = \exp(FVx/kT) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where $F = H \frac{dH}{dx} (K - K_o)$ dynes/cm^3 is the magnetic force of translation per unit volume and n_o is the concentration at x = o. For a red blood cell, in isoton saline, we may take $r = 3 \times 10^{-4}$ cm and $V = 1.1 \times 10^{-10}$ cm^3; $H \frac{dH}{dx}$ can be made as high as 10^9 oe^2/cm over a distance of 0.1 cm; $K - K_o$ is smaller than 0.2×10^{-6}, (Drake, E. N., II et al, 1963); $kT = 4.2 \times 10^{-14}$ at room temperature; x = 0.1 cm. Then,

$$F = 200 \text{ dynes/cm}^3$$

and,

$$n_o/n = e^{52500}$$

i.e., a magnetic field gradient of this size will separate red blood cells from saline very completely, if the process is allowed to go to equilibrium.

This equilibrium calculation tells us nothing about how fast equilibrium is reached. To find this out, we can use Stoke's Law. The velocity v of a sphere, in a fluid of viscosity η is given by:

$$v = \frac{FV}{6\pi\eta r} = \frac{2\, r^2 F}{9\,\eta} \quad \text{cm/sec} \quad . \quad . \quad . \quad . \quad (2)$$

using the same symbols as above. If η = 0.01 poises,

$$v = 3.9 \times 10^{-4} \text{ cms/sec or } 1.4 \text{ cm/hr}$$

for the hypothetical red blood cell discussed above. Obviously, the equilibrium separation will not be reached quickly and if the suspending fluid is moving with a velocity much larger than 1.4 cm/hr, it will be difficult to observe (S. J. Gill et al, 1960 & 1963). Most of the reported biomagnetic experiments _in vivo_ have been made in fields for which $H \frac{dH}{dx}$ is 10^{-7} or less; for them the viscosity of whole blood of 0.03 poises at 37°C would be appropriate to use; this results in a magnetic drift velocity of less than .005 cm/hr. Additionally, the value of $(K - K_o)$ chosen above is for completely reduced hemoglobin in red cells. For most cells $(K - K_o)$ would be much less than 0.2×10^{-6}. The conclusion that translational forces of gradient fields will not redistribute red blood cells under these conditions is unescapable. For smaller particles, the velocity will decrease as the square of the radius according to equation (2), so that molecular cell components, even if they were more highly magnetic, such as ferritin, are influenced even less.

As actual red blood cells are not spherical, it is of interest to see whether they could be rotated and therefore oriented in a uniform magnetic field, H, with negligible gradient $\frac{dH}{dx}$. Consider the energy U in ergs/cm^3 of a body in a magnetic field.

$$U = \frac{K_o - K}{8\pi} \int_V \vec{H^-} \cdot \vec{H_o} \, dV$$

where $\vec{H^-}$ is the field inside the body and $\vec{H_o}$ the undisturbed field. If we approximate a blood cell by an ellipsoid with semi-axes a, b, and c, then its energy, when it is oriented in a field H with a and b perpendicular to H, and c parallel to H, is given by (Stratton, J. A., 1941):

$$U_{\perp} = \frac{H^2}{1 + 4\pi K L_3} \left[\frac{abc}{6} (K_o - K) \right]$$

if H is parallel to a and perpendicular to b and c, then

$$U_{\parallel} = \frac{H^2}{1 + 4\pi K L_1} \left[\frac{abc}{6} (K_o - K) \right]$$

using the equations derived in Stratton, but in Gaussian units as above. L_1 and L_3 are given by Osborn (Osborn, J. A., 1945), who designated them L and N respectively; for a = b and $\frac{c}{a}$ = .34, L = .186, and N = .629, and

$$\Delta U = U_{\perp} - U_{\parallel} \simeq (2/3) \quad abc\, H^2 (K_o - K)\, K\, (L_1 - L_3)$$

Taking the volume (4/3) π abc = 1.1×10^{-10} cm^3 and $(K - K_o)$ = 0.2×10^{-6} as in our spherical model, with $K_o = -0.7 \times 10^{-6}$ and $H^2 = 10^8 oe^2$.

$$|\Delta U| = 2.4 \times 10^{-16} \text{ ergs}$$

As $kT = 4.2 \times 10^{-14}$ ergs at room temperature, this maximum energy difference will cause no observable rotation; a field in excess of 300,000 oersted would be needed to cause this; let us calculate the order of magnitude of the angular velocity ω radians/sec at H = 300,000 oersted.

$$\Delta U = 21.6 \times 10^{-14} \text{ erg}$$

The average torque M for a 90^o rotation will be ΔU divided by $\pi/2$; therefore,

$$M = 13.7 \times 10^{-14} \text{ dyne} \cdot \text{cm}$$

The angular velocity of a small sphere in a fluid is given (Einstein, A., 1956) by:

$$\omega = \frac{M}{8\pi\eta r^3}$$

Hence, $\omega = 0.02$ rad/sec $= 1.1^0$ per second or 68^o per minute.

This compares with the root mean square of the angular displacement $\sqrt{\overline{\theta^2}}$ due to random thermal agitation (Einstein, A., 1956).

$$\sqrt{\overline{\theta^2}} = \sqrt{t}\sqrt{\frac{kT}{4\pi\eta r^3}}$$

equal to 6.3^o in one second or 49^o in one minute in our case.

No reports of the observation of this magnetic rotation of red blood cells exist. This method of lining up the short axis of red blood cells normal to a very high magnetic field might possibly find application in the study of agglutination.

However, under the conditions of nearly all the reported biomagnetic experiments, this rotation cannot be a causative factor.

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